ACCELERATING MULTI-DOMAIN HdBNM with FAST MULTIPOLE METHOD

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Outline

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- Formulation for Multi-domain HdBNM
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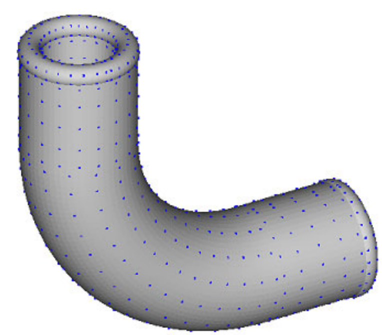
Introduction

HdBNM and FM-HdBNM

- **Hybrid Boundary Node Method**
  - Combines modified functional with the *Moving Least Squares* (MLS) approximation
  - Boundary-only truly meshless method



Example of meshless discretization

Introduction

HdBNM and FM-HdBNM

- **Fast multipole Hybrid Boundary Node Method**
  - Reducing memory requirement and total execution count to $O(N)$, where is the total number of unknowns.
  - Efficient not only in terms of human-labor costs (where mesh generation is avoided) but also in terms of computer costs, thus promising for engineering application.

Introduction

Reasons for multi-domain strategy

- **Entire domain is governed by different equations in different parts, or constructed of different materials.**
- **For better computational efficiency for long slender objects or parallel computation.**

Formulation for Multi-domain BEM

- **Domain decomposition method**

  Interface conditions are first assumed, then modified iteratively by solving individual sub-domain problems independently.
- **Standard multi-domain method**

  Equations for sub-domains are assembled into an overall system according the boundary and interface conditions.

Multi-domain HdBNM

- **HdBNM for individual sub-domains**

$$\mathbf{U}^1\mathbf{x}^1 = \mathbf{H}^1\hat{\boldsymbol{\phi}}^1 \qquad \mathbf{U}^2\mathbf{x}^2 = \mathbf{H}^2\hat{\boldsymbol{\phi}}^2 \qquad \mathbf{U}^3\mathbf{x}^3 = \mathbf{H}^3\hat{\boldsymbol{\phi}}^3$$

$$\mathbf{Q}^1\mathbf{x}^1 = \mathbf{H}^1\hat{\mathbf{q}}^1 \qquad \mathbf{Q}^2\mathbf{x}^2 = \mathbf{H}^2\hat{\mathbf{q}}^2 \qquad \mathbf{Q}^3\mathbf{x}^3 = \mathbf{H}^3\hat{\mathbf{q}}^3$$

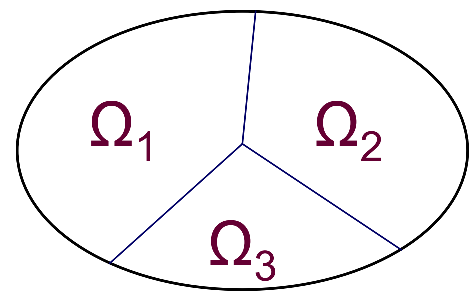
$$U_{JI} = \int_{\Gamma_{\mathbf{s}_J}} \phi_I^s(Q,\mathbf{s}_I)v_J(Q)\,d\Gamma$$

$$Q_{JI} = \int_{\Gamma_{\mathbf{s}_J}} \frac{\partial \phi_I^s(Q,\mathbf{s}_I)}{\partial n(Q)} v_J(Q)\,d\Gamma$$

$$H_{JI} = \int_{\Gamma_{\mathbf{s}_J}} \Phi_I(Q)v_J(Q)\,d\Gamma$$

$$\phi_I^s = \frac{1}{\kappa_1}\frac{1}{4\pi r(Q,\mathbf{s}_I)}$$

$$\tilde{\phi}(\mathbf{s}) = \sum_{I=1}^{N}\Phi_I(\mathbf{s})\hat{\phi}_I \qquad \tilde{q}(\mathbf{s}) = \sum_{I=1}^{N}\Phi_I(\mathbf{s})\hat{q}_I$$



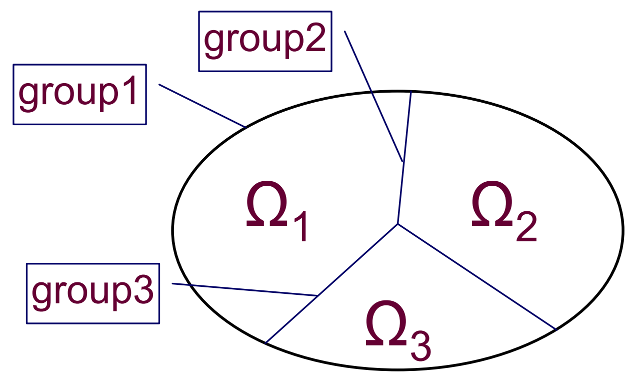
Taking three sub-domains as an example

Multi-domain HdBNM

- **Sorting unknowns according sub-domain connections**

$$\begin{bmatrix} \boldsymbol{U}_{11}^{1} & \boldsymbol{U}_{12}^{1} & \boldsymbol{U}_{13}^{1} \\ \boldsymbol{U}_{21}^{1} & \boldsymbol{U}_{22}^{1} & \boldsymbol{U}_{23}^{1} \\ \boldsymbol{U}_{31}^{1} & \boldsymbol{U}_{32}^{1} & \boldsymbol{U}_{33}^{1} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_{1}^{1} \\ \boldsymbol{x}_{2}^{1} \\ \boldsymbol{x}_{3}^{1} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{H}_{1}^{1}\hat{\boldsymbol{\phi}}_{1}^{1} \\ \boldsymbol{H}_{2}^{1}\hat{\boldsymbol{\phi}}_{2}^{1} \\ \boldsymbol{H}_{3}^{1}\hat{\boldsymbol{\phi}}_{3}^{1} \end{Bmatrix}$$

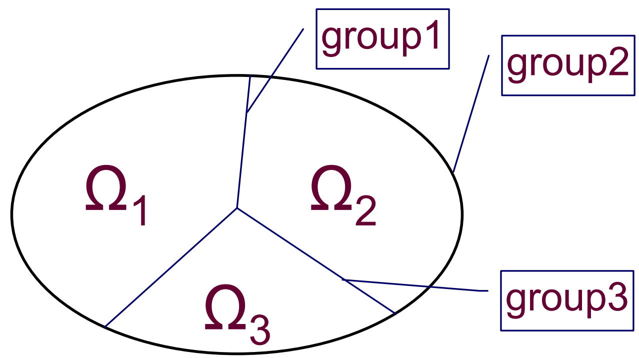
$$\begin{bmatrix} \boldsymbol{Q}_{11}^{1} & \boldsymbol{Q}_{12}^{1} & \boldsymbol{Q}_{13}^{1} \\ \boldsymbol{Q}_{21}^{1} & \boldsymbol{Q}_{22}^{1} & \boldsymbol{Q}_{23}^{1} \\ \boldsymbol{Q}_{31}^{1} & \boldsymbol{Q}_{32}^{1} & \boldsymbol{Q}_{33}^{1} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_{1}^{1} \\ \boldsymbol{x}_{2}^{1} \\ \boldsymbol{x}_{3}^{1} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{H}_{1}^{1}\hat{\boldsymbol{q}}_{1}^{1} \\ \boldsymbol{H}_{2}^{1}\hat{\boldsymbol{q}}_{2}^{1} \\ \boldsymbol{H}_{3}^{1}\hat{\boldsymbol{q}}_{3}^{1} \end{Bmatrix}$$



Multi-domain HdBNM

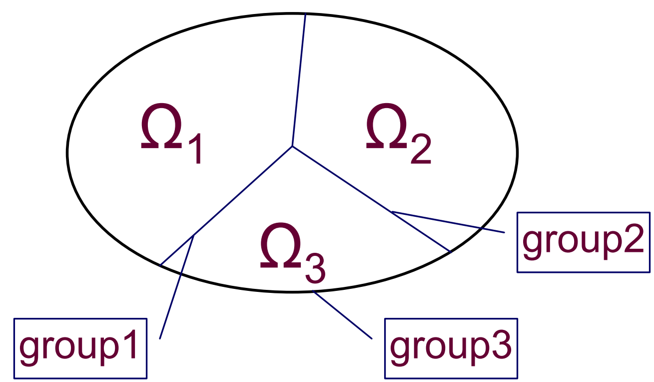
$$\begin{bmatrix} U_{11}^2 & U_{12}^2 & U_{13}^2 \\ U_{21}^2 & U_{22}^2 & U_{23}^2 \\ U_{31}^2 & U_{32}^2 & U_{33}^2 \end{bmatrix} \begin{Bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{Bmatrix} = \begin{Bmatrix} H_1^2 \hat{\phi}_1^2 \\ H_2^2 \hat{\phi}_2^2 \\ H_3^2 \hat{\phi}_3^2 \end{Bmatrix}$$

$$\begin{bmatrix} Q_{11}^2 & Q_{12}^2 & Q_{13}^2 \\ Q_{21}^2 & Q_{22}^2 & Q_{23}^2 \\ Q_{31}^2 & Q_{32}^2 & Q_{33}^2 \end{bmatrix} \begin{Bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{Bmatrix} = \begin{Bmatrix} H_1^2 \hat{q}_1^2 \\ H_2^2 \hat{q}_2^2 \\ H_3^2 \hat{q}_3^2 \end{Bmatrix}$$



$$\begin{bmatrix} U_{11}^3 & U_{12}^3 & U_{13}^3 \\ U_{21}^3 & U_{22}^3 & U_{23}^3 \\ U_{31}^3 & U_{32}^3 & U_{33}^3 \end{bmatrix} \begin{Bmatrix} x_1^3 \\ x_2^3 \\ x_3^3 \end{Bmatrix} = \begin{Bmatrix} H_1^3 \hat{\phi}_1^3 \\ H_2^3 \hat{\phi}_2^3 \\ H_3^3 \hat{\phi}_3^3 \end{Bmatrix}$$

$$\begin{bmatrix} Q_{11}^3 & Q_{12}^3 & Q_{13}^3 \\ Q_{21}^3 & Q_{22}^3 & Q_{23}^3 \\ Q_{31}^3 & Q_{32}^3 & Q_{33}^3 \end{bmatrix} \begin{Bmatrix} x_1^3 \\ x_2^3 \\ x_3^3 \end{Bmatrix} = \begin{Bmatrix} H_1^3 \hat{q}_1^3 \\ H_2^3 \hat{q}_2^3 \\ H_3^3 \hat{q}_3^3 \end{Bmatrix}$$



Multi-domain HdBNM

- **Continuity and equilibrium at the interfaces**

$$\left\{\boldsymbol{\phi}_i^j\right\} = \left\{\boldsymbol{\phi}_j^i\right\} \qquad \left\{\boldsymbol{q}_i^j\right\} = -\left\{\boldsymbol{q}_j^i\right\}$$

- **Relationship between MLS interpolation matrices at the interfaces**

$$\left\{\boldsymbol{H}_i^j\right\} = \left\{\boldsymbol{H}_j^i\right\}$$

Multi-domain HdBNM

- **Overall system of equations**

$$\begin{bmatrix} U^1_{11} & U^1_{12} & U^1_{13} \\ U^1_{21} & U^1_{22} & U^1_{23} \\ U^1_{31} & U^1_{32} & U^1_{33} \end{bmatrix} \begin{Bmatrix} x^1_1 \\ x^1_2 \\ x^1_3 \end{Bmatrix} = \begin{Bmatrix} H^1_1\hat{\phi}^1_1 \\ H^1_2\hat{\phi}^1_2 \\ H^1_3\hat{\phi}^1_3 \end{Bmatrix} \qquad \begin{bmatrix} Q^1_{11} & Q^1_{12} & Q^1_{13} \\ Q^1_{21} & Q^1_{22} & Q^1_{23} \\ Q^1_{31} & Q^1_{32} & Q^1_{33} \end{bmatrix} \begin{Bmatrix} x^1_1 \\ x^1_2 \\ x^1_3 \end{Bmatrix} = \begin{Bmatrix} H^1_1\hat{q}^1_1 \\ H^1_2\hat{q}^1_2 \\ H^1_3\hat{q}^1_3 \end{Bmatrix}$$

$$\begin{bmatrix} U^2_{11} & U^2_{12} & U^2_{13} \\ U^2_{21} & U^2_{22} & U^2_{23} \\ U^2_{31} & U^2_{32} & U^2_{33} \end{bmatrix} \begin{Bmatrix} x^2_1 \\ x^2_2 \\ x^2_3 \end{Bmatrix} = \begin{Bmatrix} H^2_1\hat{\phi}^2_1 \\ H^2_2\hat{\phi}^2_2 \\ H^2_3\hat{\phi}^2_3 \end{Bmatrix}$$

$$\begin{bmatrix} Q^2_{11} & Q^2_{12} & Q^2_{13} \\ Q^2_{21} & Q^2_{22} & Q^2_{23} \\ Q^2_{31} & Q^2_{32} & Q^2_{33} \end{bmatrix} \begin{Bmatrix} x^2_1 \\ x^2_2 \\ x^2_3 \end{Bmatrix} = \begin{Bmatrix} H^2_1\hat{q}^2_1 \\ H^2_2\hat{q}^2_2 \\ H^2_3\hat{q}^2_3 \end{Bmatrix}$$

$$\begin{bmatrix} A^1_{11} & A^1_{12} & A^1_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ U^1_{21} & U^1_{22} & U^1_{23} & -U^2_{11} & -U^2_{12} & -U^2_{13} & 0 & 0 & 0 \\ U^1_{31} & U^1_{32} & U^1_{33} & 0 & 0 & 0 & -U^3_{11} & -U^3_{12} & -U^3_{13} \\ Q^1_{21} & Q^1_{22} & Q^1_{23} & Q^2_{11} & Q^2_{12} & Q^2_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & A^2_{21} & A^2_{22} & A^2_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & U^2_{31} & U^2_{32} & U^2_{33} & -U^3_{21} & -U^3_{22} & -U^3_{23} \\ Q^1_{31} & Q^1_{32} & Q^1_{33} & 0 & 0 & 0 & Q^3_{11} & Q^3_{12} & Q^3_{13} \\ 0 & 0 & 0 & Q^2_{31} & Q^2_{32} & Q^2_{33} & Q^3_{21} & Q^3_{22} & Q^3_{23} \\ 0 & 0 & 0 & 0 & 0 & 0 & A^3_{31} & A^3_{32} & A^3_{33} \end{bmatrix} \begin{Bmatrix} x^1_1 \\ x^1_2 \\ x^1_3 \\ x^2_1 \\ x^2_2 \\ x^2_3 \\ x^3_1 \\ x^3_2 \\ x^3_3 \end{Bmatrix} = \begin{Bmatrix} d^1_1 \\ 0 \\ 0 \\ 0 \\ d^2_2 \\ 0 \\ 0 \\ 0 \\ d^3_3 \end{Bmatrix}$$

$$\begin{bmatrix} U^3_{11} & U^3_{12} & U^3_{13} \\ U^3_{21} & U^3_{22} & U^3_{23} \\ U^3_{31} & U^3_{32} & U^3_{33} \end{bmatrix} \begin{Bmatrix} x^3_1 \\ x^3_2 \\ x^3_3 \end{Bmatrix} = \begin{Bmatrix} H^3_1\hat{\phi}^3_1 \\ H^3_2\hat{\phi}^3_2 \\ H^3_3\hat{\phi}^3_3 \end{Bmatrix} \qquad \begin{bmatrix} Q^3_{11} & Q^3_{12} & Q^3_{13} \\ Q^3_{21} & Q^3_{22} & Q^3_{23} \\ Q^3_{31} & Q^3_{32} & Q^3_{33} \end{bmatrix} \begin{Bmatrix} x^3_1 \\ x^3_2 \\ x^3_3 \end{Bmatrix} = \begin{Bmatrix} H^3_1\hat{q}^3_1 \\ H^3_2\hat{q}^3_2 \\ H^3_3\hat{q}^3_3 \end{Bmatrix}$$

Accelerating Multi-domain HdBNM by FMM

$$\begin{bmatrix} \boldsymbol{A}_{11}^1 & \boldsymbol{A}_{12}^1 & \boldsymbol{A}_{13}^1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{U}_{21}^1 & \boldsymbol{U}_{22}^1 & \boldsymbol{U}_{23}^1 & -\boldsymbol{U}_{11}^2 & -\boldsymbol{U}_{12}^2 & -\boldsymbol{U}_{13}^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{U}_{31}^1 & \boldsymbol{U}_{32}^1 & \boldsymbol{U}_{33}^1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{U}_{11}^3 & -\boldsymbol{U}_{12}^3 & -\boldsymbol{U}_{13}^3 \\ \boldsymbol{Q}_{21}^1 & \boldsymbol{Q}_{22}^1 & \boldsymbol{Q}_{23}^1 & \boldsymbol{Q}_{11}^2 & \boldsymbol{Q}_{12}^2 & \boldsymbol{Q}_{13}^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{21}^2 & \boldsymbol{A}_{22}^2 & \boldsymbol{A}_{23}^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{U}_{31}^2 & \boldsymbol{U}_{32}^2 & \boldsymbol{U}_{33}^2 & -\boldsymbol{U}_{21}^3 & -\boldsymbol{U}_{22}^3 & -\boldsymbol{U}_{23}^3 \\ \boldsymbol{Q}_{31}^1 & \boldsymbol{Q}_{32}^1 & \boldsymbol{Q}_{33}^1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{Q}_{11}^3 & \boldsymbol{Q}_{12}^3 & \boldsymbol{Q}_{13}^3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{Q}_{31}^2 & \boldsymbol{Q}_{32}^2 & \boldsymbol{Q}_{33}^2 & \boldsymbol{Q}_{21}^3 & \boldsymbol{Q}_{22}^3 & \boldsymbol{Q}_{23}^3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{31}^3 & \boldsymbol{A}_{32}^3 & \boldsymbol{A}_{33}^3 \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_1^1 \\ \boldsymbol{x}_2^1 \\ \boldsymbol{x}_3^1 \\ \boldsymbol{x}_1^2 \\ \boldsymbol{x}_2^2 \\ \boldsymbol{x}_3^2 \\ \boldsymbol{x}_1^3 \\ \boldsymbol{x}_2^3 \\ \boldsymbol{x}_3^3 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{d}_1^1 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{d}_2^2 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{d}_3^3 \end{Bmatrix}$$

- **FMM is an algorithm for achieving fast products of dense matrices with vectors in the methods based on fundamental solutions.**
- **We have implemented the FMM techniques in the HdBNM for single domain problems.**
- **We divide the matrix-vector product into smaller ones at the subdomain level.**

Accelerating Multi-domain HdBNM by FMM

- **For an iteration vector x´, we suppose, in individual sub-domains,**

$$\begin{bmatrix} \boldsymbol{U}_{11}^i & \boldsymbol{U}_{12}^i & \boldsymbol{U}_{13}^i \\ \boldsymbol{U}_{21}^i & \boldsymbol{U}_{22}^i & \boldsymbol{U}_{23}^i \\ \boldsymbol{U}_{31}^i & \boldsymbol{U}_{32}^i & \boldsymbol{U}_{33}^i \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_1'^i \\ \boldsymbol{x}_2'^i \\ \boldsymbol{x}_3'^i \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\phi}_1^i \\ \boldsymbol{\phi}_2^i \\ \boldsymbol{\phi}_3^i \end{Bmatrix} \qquad \begin{bmatrix} \boldsymbol{Q}_{11}^i & \boldsymbol{Q}_{12}^i & \boldsymbol{Q}_{13}^i \\ \boldsymbol{Q}_{21}^i & \boldsymbol{Q}_{22}^i & \boldsymbol{Q}_{23}^i \\ \boldsymbol{Q}_{31}^i & \boldsymbol{Q}_{32}^i & \boldsymbol{Q}_{33}^i \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_1'^i \\ \boldsymbol{x}_2'^i \\ \boldsymbol{x}_3'^i \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{q}_1^i \\ \boldsymbol{q}_2^i \\ \boldsymbol{q}_3^i \end{Bmatrix}$$

- **The overall matrix-vector product can be obtained:**

$$\begin{bmatrix} \boldsymbol{A}_{11}^1 & \boldsymbol{A}_{12}^1 & \boldsymbol{A}_{13}^1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{U}_{21}^1 & \boldsymbol{U}_{22}^1 & \boldsymbol{U}_{23}^1 & -\boldsymbol{U}_{11}^2 & -\boldsymbol{U}_{12}^2 & -\boldsymbol{U}_{13}^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{U}_{31}^1 & \boldsymbol{U}_{32}^1 & \boldsymbol{U}_{33}^1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{U}_{11}^3 & -\boldsymbol{U}_{12}^3 & -\boldsymbol{U}_{13}^3 \\ \boldsymbol{Q}_{21}^1 & \boldsymbol{Q}_{22}^1 & \boldsymbol{Q}_{23}^1 & \boldsymbol{Q}_{11}^2 & \boldsymbol{Q}_{12}^2 & \boldsymbol{Q}_{13}^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{21}^2 & \boldsymbol{A}_{22}^2 & \boldsymbol{A}_{23}^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{U}_{31}^2 & \boldsymbol{U}_{32}^2 & \boldsymbol{U}_{33}^2 & -\boldsymbol{U}_{21}^3 & -\boldsymbol{U}_{22}^3 & -\boldsymbol{U}_{23}^3 \\ \boldsymbol{Q}_{31}^1 & \boldsymbol{Q}_{32}^1 & \boldsymbol{Q}_{33}^1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{Q}_{11}^3 & \boldsymbol{Q}_{12}^3 & \boldsymbol{Q}_{13}^3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{Q}_{31}^2 & \boldsymbol{Q}_{32}^2 & \boldsymbol{Q}_{33}^2 & \boldsymbol{Q}_{21}^3 & \boldsymbol{Q}_{22}^3 & \boldsymbol{Q}_{23}^3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{31}^3 & \boldsymbol{A}_{32}^3 & \boldsymbol{A}_{33}^3 \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_1'^1 \\ \boldsymbol{x}_2'^1 \\ \boldsymbol{x}_3'^1 \\ \boldsymbol{x}_1'^2 \\ \boldsymbol{x}_2'^2 \\ \boldsymbol{x}_3'^2 \\ \boldsymbol{x}_1'^3 \\ \boldsymbol{x}_2'^3 \\ \boldsymbol{x}_3'^3 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\phi}_1^1 \text{ or } \boldsymbol{q}_1^1 \\ \boldsymbol{\phi}_2^1 - \boldsymbol{\phi}_1^2 \\ \boldsymbol{\phi}_3^1 - \boldsymbol{\phi}_1^3 \\ \boldsymbol{q}_2^1 + \boldsymbol{q}_1^2 \\ \boldsymbol{\phi}_2^2 \text{ or } \boldsymbol{q}_2^2 \\ \boldsymbol{\phi}_3^2 - \boldsymbol{\phi}_2^3 \\ \boldsymbol{q}_3^1 + \boldsymbol{q}_1^3 \\ \boldsymbol{q}_3^2 + \boldsymbol{q}_2^3 \\ \boldsymbol{\phi}_3^3 \text{ or } \boldsymbol{q}_3^3 \end{Bmatrix}$$

Accelerating Multi-domain HdBNM by FMM

- **Remarks:**

$$\begin{bmatrix} A^1_{11} & A^1_{12} & A^1_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ U^1_{21} & U^1_{22} & U^1_{23} & -U^2_{11} & -U^2_{12} & -U^2_{13} & 0 & 0 & 0 \\ U^1_{31} & U^1_{32} & U^1_{33} & 0 & 0 & 0 & -U^3_{11} & -U^3_{12} & -U^3_{13} \\ Q^1_{21} & Q^1_{22} & Q^1_{23} & Q^2_{11} & Q^2_{12} & Q^2_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & A^2_{21} & A^2_{22} & A^2_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & U^2_{31} & U^2_{32} & U^2_{33} & -U^3_{21} & -U^3_{22} & -U^3_{23} \\ Q^1_{31} & Q^1_{32} & Q^1_{33} & 0 & 0 & 0 & Q^3_{11} & Q^3_{12} & Q^3_{13} \\ 0 & 0 & 0 & Q^2_{31} & Q^2_{32} & Q^2_{33} & Q^3_{21} & Q^3_{22} & Q^3_{23} \\ 0 & 0 & 0 & 0 & 0 & 0 & A^3_{31} & A^3_{32} & A^3_{33} \end{bmatrix} \begin{Bmatrix} x^1_1 \\ x^1_2 \\ x^1_3 \\ x^2_1 \\ x^2_2 \\ x^2_3 \\ x^3_1 \\ x^3_2 \\ x^3_3 \end{Bmatrix} = \begin{Bmatrix} d^1_1 \\ 0 \\ 0 \\ 0 \\ d^2_2 \\ 0 \\ 0 \\ 0 \\ d^3_3 \end{Bmatrix}$$

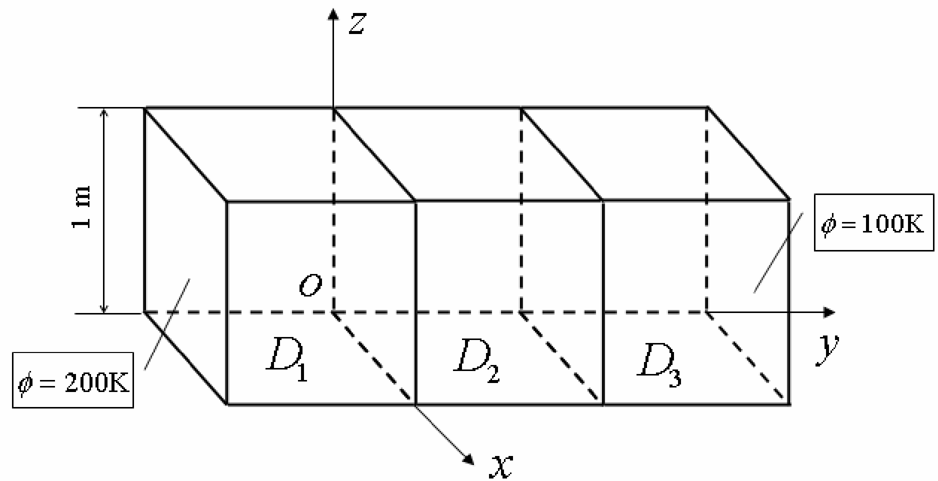
- The matrix needs never be formed, it is purely symbolic.
- The sparsity pattern has a severe impact on the condition number of the matrix. The order of unknowns is determined by listing all permutations of two sub-domains:

  11 12 13 21* 22 23 31* 32* 33

- As a first step, we use a block diagonal preconditioner.

Numerical examples

- **Three cubes**

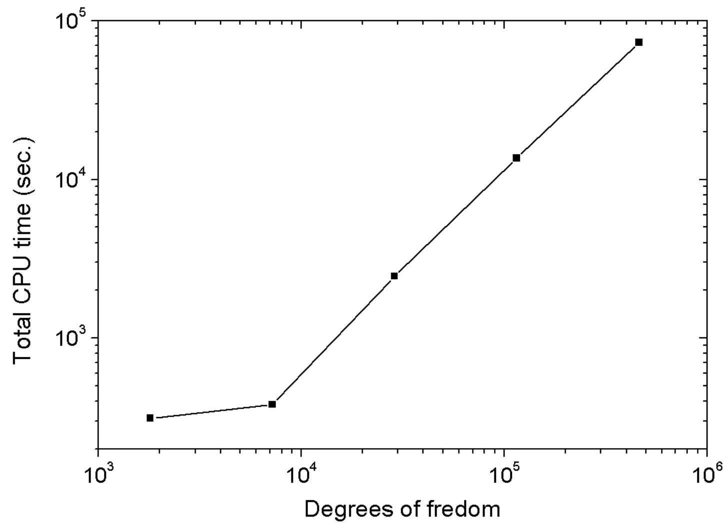


κ_1=1.0W/mK, κ_2=3.0W/mK, κ_3=2.0W/mK

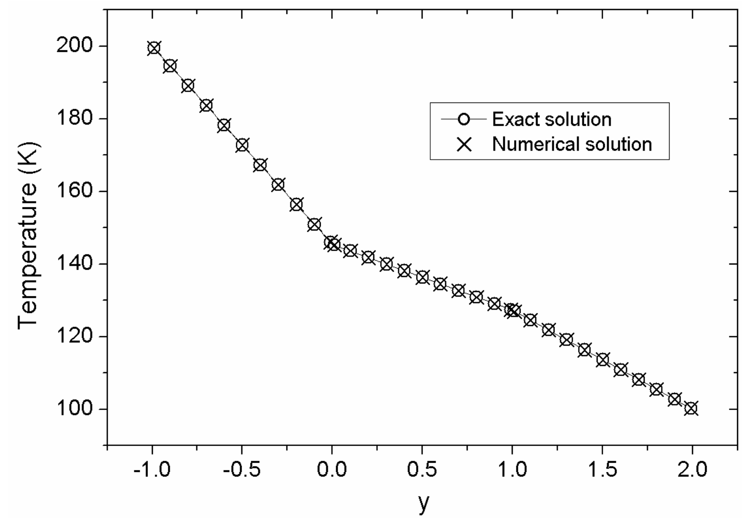
Exact solution:

$$\phi = \begin{cases} (1600\text{-}600\times y)/11, & -1 \le y < 0 \\ (1600\text{-}200\times y)/11, & 0 \le y < 1 \\ (1700\text{-}300\times y)/11, & 1 \le y \le 2 \end{cases}$$

Numerical examples



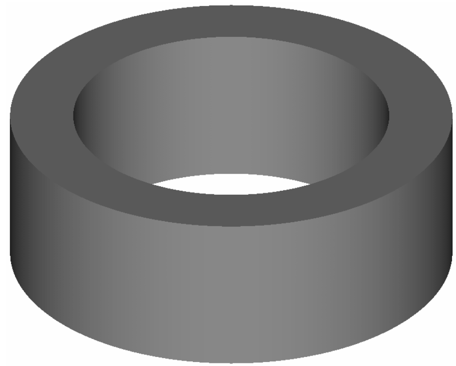
Total CPU time vs. DOFs



Temperature distribution

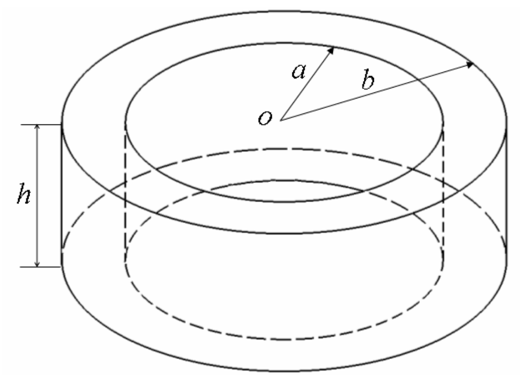
Numerical examples

- **A cylinder**

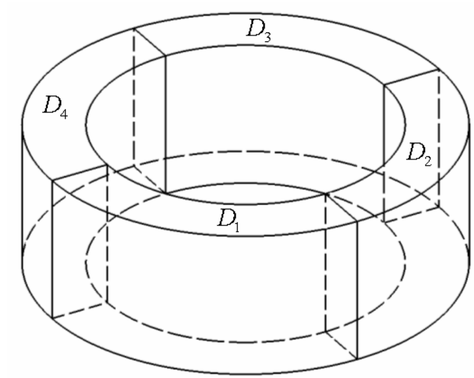


Analytical solution:

$$\phi = x^3 + y^3 + z^3 - 3yx^2 - 3xz^2 - 3zy^2$$

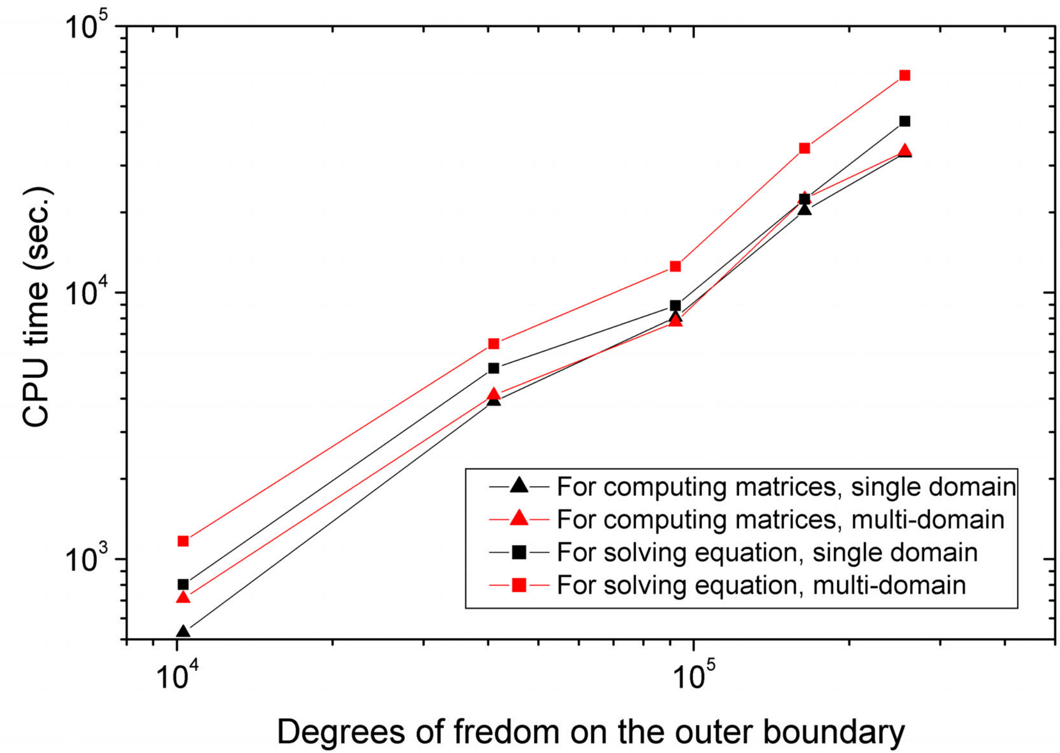


Single domain model



Four sub-domains model

Numerical examples



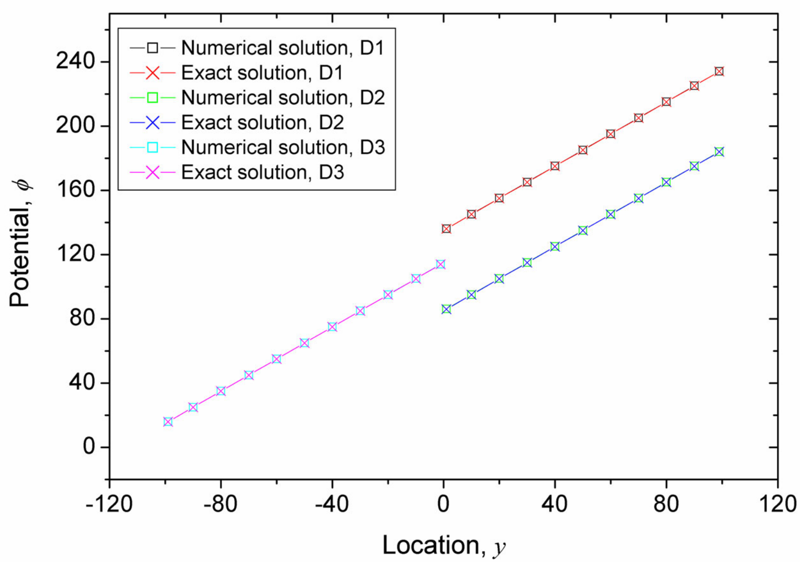
Timing results for the thick cylinder problem

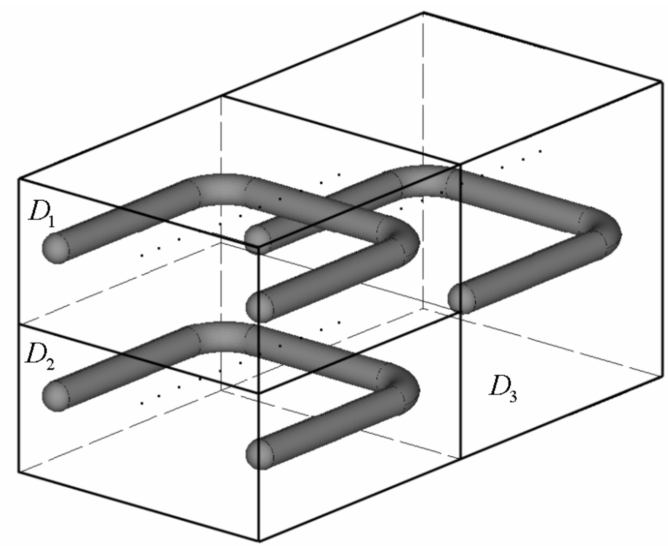
Numerical examples

- **Three boxes with inclusions**

Analytical solution: $\phi = x + y + z$

Total number of unknowns: 57506





Conclusions

- The FMM techniques have been implemented in the multi-domain HdBNM for solving potential problems. High accuracy and efficiency have been achieved.
- In contrast to conventional multi-domain BEM, multi-domain strategies can not be used to get better computational efficiency for long slender objects in FMM context, because the FMM has already reduced the computational scale to nearly linear complexity
- Developing other preconditioners especially suitable for multi-domain HdBNM in the FMM context is an ongoing research.